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Kurt Gödel. Das Album—The Album

Karl Sigmund, John Dawson, and Kurt Mühlberger, with a Foreword by Hans Magnus Enzensberger

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REVIEWED BY JULIETTE KENNEDY

The Austrian logician Kurt Gödel, a brilliant man beset by misfortunes of all kinds, has drawn the attention of a number of biographers—with mixed results. One wonders about the rights of the deceased in such matters. These lines from Othello's final soliloquy (from Act 5 of Shakespeare's play) seem quite apt:

> I have done the state some service and they know't.
> No more of that. I pray you, in your letters,
> When you shall these unlucky deeds relate,
> Speak of me as I am. Nothing extenuate,
> Nor aught set down in malice.

Such being the case, *Kurt Gödel. The Album* comes as a welcome addition to the literature on Gödel. As the catalogue of the exhibition which took place at the University of Vienna this year in conjunction with the celebration there on the occasion of Gödel's centennial, it is to a large extent made up of reproductions of documents and photographs with accompanying commentary. And while far too many of the letters and other documents are undated, compromising the historical value of the volume, it is nevertheless one of the most fascinating and also one of the most corrective books to appear about Gödel so far.

The book is divided into three sections: a biographical section comprising roughly half of the book, a section devoted to Gödel's work, a brief section titled "Gödel's Vienna." These are followed by an appendix consisting of two synopses of Gödel's 1931 incompleteness theorems, the *Erkenntnis* piece of 1931 due to Gödel himself, and the text of a lecture delivered in Vienna in 1932 by Karl Menger. And although the authors write in the introduction that the book is meant as an "easily digestible introduction" to Gödel's life, work, and the Viennese culture in which he lived and flourished, it will undoubtedly be of interest to readers on every level, from newcomers to this area of the history of mathematics to seasoned logicians and mathematicians.

Gödel's Life

"Gödel in a good mood and brilliant as usual. . . . I like him infinitely much and no one, no one of my friends can stimulate me as he does." So wrote Gödel's friend Oskar Morgenstern after visiting with Gödel, presumably sometime in the mid-1970s.

A number of interesting documents are included in this first section. For example, a few pages from one of Gödel's schoolbooks are reproduced on page 18. They are undated, but Gödel must have been a very young child when he wrote these lines. In a typical example of the old-fashioned rote method of teaching, in the workbook Gödel had to write the symbols "+" and the numbers "2" and "3" a hundred (or more) times, presumably for practice, and then finally the equations "1 + 1 = 2" and "2 + 1 = 3." But then his mind wandered and he wrote "1+ = 21." His report card of February 10, 1917 is here too, on page 19, and there we see that the 10-year-old boy rated "sehr gut" in every subject—except mathematics!, in which he only rated a "gut."

A fascinating set of pictures and captions deal with Gödel's first few years in Vienna, where he became a student at the University of Vienna in 1924. It seems to have been a golden time for Gödel, intellectually and in other ways too. One could hardly walk down the street in Vienna in the 1920s without bumping into a major cultural or scientific luminary of the twentieth century—and the University of Vienna was no less blessed in this respect. Gödel attended lectures by Philipp Furtwängler (cousin of the great conductor), Heinrich Gomperz, Hans Hahn, Moritz Schlick, and Rudolf Carnap (all of whom are pictured here), to name just a few; he also quickly found for himself a society of like-minded and, judging from the photographs, high-living, university friends, who, as was customary then in Vienna, spent their days in a series of roving discussions in cafes, some of which are pictured. Many of these friends were to go on to prominent positions in post-war academia: Herbert Feigl, for example, a student of Moritz Schlick and one of Gödel's best friends, was to become president of the American Philosophical Association—but he is seen here as a young man cavorting at the beach with the Schlick family. Other friends from this period who would later emigrate to the United States include Karl Menger, Olga Taussky, and Oskar Morgenstern.

Schlick, a central figure in the Viennese philosophical culture at the time, was, together with Hans Hahn, the leader of the so-called Vienna Circle, also called the Schlick Circle, a discussion group Gödel attended that quickly became identified with the doctrine of "logical positivism" (a term coined by Feigl and Blumberg in their 1931 "Logical positivism: A new movement in European philosophy" [1]). Gödel was never drawn to logical positivism himself, but the exposure to the discussions that took place at those meetings must have been crucial to his development as a logician. Otto Neurath, Karl Menger, Gustav Bergmann, Rudolf Carnap, and Friedrich Waismann were just a few of the regular members; Ludwig Wittgenstein visited periodically, as did outsiders like Frank Ramsey and W.V.O. Quine.

That Hahn and Schlick would come together to lead this historic seminar was characteristic of the hybrid nature of logic at the time. Gödel commented on this to Hao Wang in the late 1970s ([4], p. 82):

> When I entered the field of logic, there were fifty percent philosophy and fifty percent mathematics. There are now ninety percent mathematics and only one percent philosophy . . .

Gödel began his studies in Vienna in 1924 in physics with Hans Thirring; and although he switched to mathe-

> ***The sun was setting on this golden period of Gödel's life***

matics two years later, studying first number theory under Furtwängler and then logic under Hahn and, informally, Carnap, he continued to take physics courses up to the time he graduated in 1929. The background in physics Gödel obtained as a student may explain his being able to make a significant contribution to relativity in 1947. (See [2].) In fact, Gödel wrote this about his work in relativity in a 1955 letter to Carl Seelig (see [3], p. 252):

> I have, however, in connection with certain philosophical problems, devoted myself for some time to a less difficult complex of questions from general relativity theory, namely cosmology. The fact that here I, as a newcomer to the field of relativity theory, could immediately obtain essentially new results seems to me sufficient to demonstrate the unfinished state of the theory.

That the sun was setting on this golden period of Gödel's, if not Vienna's, if not indeed Europe's existence, is first alluded to on page 26, where it is mentioned that when Gödel obtained his Austrian citizenship in 1929, the year he turned in his thesis containing the completeness theorem for first-order logic, the political situation in Vienna was becoming increasingly violent, with riots in 1927 leaving 89 people dead and the conservatives in essentially open war with the socialist majority. The political situation manifested itself with particular vehemence at the University of Vienna, which, being considered particularly "red," was increasingly targeted by the authorities—with, of course, disastrous results as the 1930s wore on and National Socialism eventually saturated the university. By May of 1938, out of 258 emeritus and full professors, associate professors, and lecturers, 94 had "retired" from the philosophical faculty; and those salaried employees who wished to remain had to endure a so-called dejudification procedure in order to keep their positions.

The oath to the Führer that university personnel had to sign is reproduced here on page 57, as is a letter from the Studentenführer from that month on page 56, apprising students of the new post-Anschluss decorum, which required them to stand and give the Hitler salute at the beginning and end of all lectures. Such an occasion is pictured in a chilling photograph on page 55 of the book.

Finally, in a tragic symptom of the times, Schlick was assassinated at the university in 1936 by an ex-student named Nelböck, whom Gödel knew from his mathematics and philosophy classes. After mounting a defense based on the idea that Schlick's atheism had caused him to become deranged, Nelböck got off after serving only two years for the crime. In a post-war coda to the Nelböck episode, we learn (on page 186) that Viktor Kraft, who wrote the first book about the Vienna Circle, was successfully sued by Nelböck in 1950, for Kraft's use of the term "persecution mania" to describe Nelböck at the time of the assassination—that is to say, Kraft had to withdraw the expression. Needless to say, deprived of one of its founding members, the Vienna Circle broke up soon afterwards.

The thought of Gödel in the midst of all this gives one pause, although of course, not being Jewish, his difficulties paled besides those of his Jewish friends and teachers. Nevertheless Gödel's difficulties during the 1930s—his most productive decade mathematically—were substantial, and are documented here. He was hospitalized a number of times with nervous breakdowns, due perhaps not only to a natural inclination in this direction, but perhaps in part also due to outside circumstances. Hahn as well as Schlick had died, and his Dozentur, which he had gotten in 1932, was withdrawn and could only be restored as a so-called "Dozent neuer Ordnung" upon his being found ethnically and politically acceptable to the Nazi regime. At the same time his problems with the authorities were compounded by his three trips back and forth to the United States during that decade, each of which required a travel visa. Finally, he was roughed up in the street one day by some young men, reportedly members of the Hitler Youth.

Regarding Gödel's university position, in a letter to the Rector (reproduced here on page 68), the Ministry of Interior and Cultural Affairs recommended against Gödel being granted the neue Dozentur, on the grounds of his "doubtful" political stance. This was not just an idle guess on the part of the Ministry, but the result of an investigation involving the interrogation of Gödel's friends and colleagues. Nevertheless, Gödel was eventually granted the Dozentur in 1940; but by that time he and his wife Adele had already moved to the United States. The Gödels were never to return to Europe. In fact Gödel would write to his mother (in a letter reproduced on page 87) that for a time he was plagued by nightmares of being trapped in Vienna and not being able to leave.

As for Gödel's life in the United States, one letter stands out particularly as a footnote to the Gödel-Thirring connection. Before Gödel left Vienna for the United States in 1940, Thirring asked him to warn Einstein that Nazi Germany might be in a position to develop a nuclear weapon (the possibility of atomic fission having been discovered in Berlin in late December 1938, when for the first time fission was actually produced in the laboratory). Many years later (in 1972) Thirring wrote to Gödel and asked him if he had ever passed on the warning. In this letter (reproduced on page 143) Gödel replied that he had not passed it on. He gave a number of reasons: at the time he had been out of contact with physics for a decade, and hadn't known about the development of fission; but then when he did finally hear about "these things," he was skeptical—not for any scientific reason, but for what he called a "sociological reason": he didn't think the culture at the time was at such a point in its evolution for such a development to ensue. Rather this should come at the end of our "Kulturperiode."

Although it is not difficult to imagine Gödel's real reasons for not contacting Einstein about Thirring's warning—certainly the reasons Gödel gives here do not seem very convincing—one may nevertheless be strongly inclined to take Gödel to task over the matter. As it happens the physicist Leo Szilard had already visited Einstein in 1939 to convey the same warning. In any case, after the war Gödel's and Einstein's views about the stockpiling of nuclear weapons and the pursuit of the ensuing arms race between the USSR and the United States, and about matters of war and peace in general, were nearly indistinguishable. Gödel had no hesitation about expressing those views—at least in the 1950s:

> Einstein warned the world not to try to attain peace by rearmament and intimidating the adversaries. He said that this procedure would lead to war and not to peace, and he was quite right. And the fact is well known that the other procedure (trying to come to an agreement in an amiable way) wasn't even attempted by America, but refused from the first. It isn't the one and only question as to who started matters, and for the most part it would be difficult to establish. But one thing is certain: under the slogan "democracy," America is waging a war for an absolutely unpopular regime and under the name of a "police action" for the UN and does things to which even the UN does not agree . . .

Gödel's correspondence with his mother was being monitored by the FBI, presumably because of Gödel's friendship with Einstein. Accordingly this passage of Gödel's 1950 letter was included in a report written by one General Cornelius Moynihan of the U.S. armed forces, to none other than J. Edgar Hoover (see page 145).

Gödel's Work

In this section of the book one sees most clearly that it is aimed at a general audience; the expert may take issue with the finer points of the descriptions of Gödel's theorems or of his philosophical work, or find those accounts too sparse. But the mathematics here is in all particulars completely correct. Also the historical emphasis is right: Hilbert, Russell, Tarski, von Neumann, Turing—all are included, with no small amount of text devoted to their contributions to logic. As is Cantor—indeed a page from a letter he wrote to Hilbert is included here, written in Cantor's extravagantly florid handwriting. We also meet up with Husserl, and Leibniz, and even Goldbach. In pages headed "Time travel with Gödel," Gödel's work in relativity is touched upon, with a long quote from Palle Yourgrau summarizing Gödel's work and why it implies that "one can travel to any region of the past."

To show how Gödel's work in physics was seen by contemporary physicists, the authors include a note from the then Director of the Institute for Advanced Study, Harry Woolf (to himself, presumably) listing the topics to be spoken about at Gödel's funeral, which took place in 1978:

1. set theory + the continuum hypothesis
2. logic = incompleteness + consistency
3. (Minor): relativity—not worth a talk—X.

On the opposite end of the spectrum, so to speak, the subsection called "Theology" begins somewhat inauspiciously with an undated notebook from the Gödel Nachlass titled: "Errors in the Bible." But then it goes on to discuss Gödel's ontological proof and his wider views on exact theology in a very sensible way.

One of the most interesting documents in the book appears in this section (page 159): the receipt for the copy of the papal encyclical called "Mit brennender Sorge" (in English "With burning worry") issued by Pope Pius XI.[1] The encyclical, published (unusually) in German in March of 1937 but distributed secretly, condemns Nazism. The receipt indicates that Gödel purchased his copy on December 20, 1937.

Unfortunately, *very* short shrift is given to Gödel's philosophical work. For example, in a section called "Plato's Shadow," subtitled "An unadulterated Platonist,"[2] we are only told that Gödel's Platonist views "appear strange in the twentieth century."

Gödel's Vienna

Much to their credit, the authors cast a wide net in this section, including not only photographs and text devoted to Gödel's scientific milieu, but to figures from the wider culture such as Robert Musil, whom Gödel never met, and Hermann Broch, whom he did. Interestingly, both wrote novels whose heroes are mathematicians; in fact, the authors remind us on page 194 that Broch's 1933 novel called *The Unknown Quantity* features a hero who dreams of finding a logic without axioms—not a little reminiscent of the project of informal rigor due to Gödel and also to Georg Kreisel.

The book ends with two letters the poet Hans Magnus Enzensberger wrote to Gödel in 1957 and in 1974, asking him for an interview. I was delighted to notice that the return address on the 1974 letter is 15 Commerce Street, New York City, which was then the home of Christiane Zimmer, *née* von Hofmannstahl, daughter of the famous poet and Strauss librettist Hugo von Hofmannstahl, and herself a wartime emigré to the United States with her husband the Indianologist Heinrich Zimmer. Most likely Enzensberger would have been staying with Mrs. Zimmer, who by 1974 had become the *sine qua non* of emigré German and Austrian life in New York City: she offered material and other support to untold numbers of artists—for example, the food served at her Sunday night salons, which I had the pleasure of attending in its last years, was often the only square meal an artist could have gotten in New York that week. She also offered hospitality to any and all visiting German and Austrian writers, such as Max Frisch, Günter Grass, and Siegfried Lenz.

Her picture should perhaps have been included here; not necessarily for her very faint connection to Gödel, but because she demonstrated so beautifully the idea that one could create a new and vibrant life in a new country, even after being forced out of one's *own* country—a lesson Gödel was never to learn, scarred as he was by the events of the 1930s, and perhaps by events from his childhood too.

REFERENCES

[1] Herbert Feigl and Albert Blumberg, Logical positivism. a new movement in European philosophy. *Journal of Philosophy* 28:281–296, 1931.

[2] Kurt Gödel, "An example of a new type of cosmological solutions of Einstein's field equations of gravitation," *Reviews of modern physics* 21:447–450, 1949.

[3] Kurt Gödel, *Collected Works. V: Correspondence H-Z.* S. Feferman, *et al.*, eds., Oxford University Press, Oxford, 2003.

[1] In a rare slip, the authors get the German title wrong, misidentify it as a bull, and attribute it to Pius XII.

[2] This is actually Russell's description of Gödel in the second volume of Russell's *Autobiography*, on the occasion of his meeting Gödel at Einstein's house in Princeton in 1943.

[4] Hao Wang, *A logical journey: From Gödel to philosophy*. Final edition and with an addition to the preface by Palle Yourgrau and Leigh Cauman. MIT Press, Cambridge, MA, 1996.

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Die Vermessung des Unendlichen (Measuring Infinity)

an opera by Ingomar Grünauer

REVIEWED BY JEAN-MICHEL KANTOR

Part of mathematics consists in giving precise meanings to specific words. Take 'infinity': it first appeared two thousand years ago when Anaximander of Miletus, a pre-Socratic philosopher, coined the word 'Apeiron'—an obscure notion of vagueness and unboundedness.[1] It took centuries to give a mathematical meaning to infinity. And it is still not over![2] The first, crucial step was Aristotle's claim that there was no 'actual infinity', just the 'potential infinity' represented by one, two, three, . . . In fact, until recent surprising discoveries concerning Archimedes,[3] general opinion held that for the Ancient Greeks there was no real infinity.

The mathematics of infinity in the modern sense began in the middle of the nineteenth century with the priest Bernard Bolzano (1781–1848). Bolzano wrote a book, *Paradoxes of the Infinite*, in which he tried to define a calculus with all sorts of infinities (infinitely small and large). When Georg Cantor (1845–1918), studying the sets of unicity (now called exceptional sets) of trigonometric series, extended the work of his colleague Heinrich Heine (1821–1881), he was led naturally to the study of parts of the continuum, the set of all real numbers. Cantor used the language of sets and functions, and with his proof that the set of real numbers is non-denumerable he set the stage for the new mathematical theory of infinity. Then he introduced the main actors: transfinite numbers, cardinals and ordinals, and the Alephs. But Cantor was also hoping to achieve a sublime goal—understanding the infinity of infinities. Not only was he aware of the religious dimension of his work, it was a strong stimulus, as it had been for other mathematicians, such as Pythagoras, or Luzin and the Name-worshippers, or Gödel. Cantor considered his theory of sets a revelation of truth inspired by God.

Much later, set theory became the *lingua franca* of mathematics, and although some important difficulties had been apparent from the beginning, the new math (as they called it) was taught in most schools from the 1960s on.

Cantor, who created set theory, was born in Saint Petersburg, but moved with his family to Germany when he was twelve. He was a professor at Halle University from 1869 to 1913.[4] Cantor suffered early from manic depression, which increased after 1899, and he later spent periods in various psychiatric institutions. He is buried in Halle.

The city of Halle was founded 1200 years ago, but it has recently fallen on hard times: due to the economic crisis in the former German Democratic Republic, the city lost a third of its population in five years. The city center has many old and well-preserved buildings, among them the house where Georg Friedrich Händel was born, but the outer parts of the city resemble a Russian provincial town. The Opernhaus of Halle is a nice building of classical eighteenth-century style.

For its Jubilaeum, the city council, to honor Cantor, the other of its two great men, commissioned an opera from Ingomar Grünauer, a composer born in Vienna in 1938. Grünauer did not try to give a precise mathematical account of Cantor's work, but rather was loosely inspired by

–the resistance from the older generation of mathematicians to the new mathematics,
–the resistance from younger elements like Cantor's friend Schwarz,
–the efforts of Cantor to prove the Continuum Hypothesis (CH), one of the central questions of set theory even now, and
–Cantor's mental problems and problems connected with his wife and daughter.

On stage Cantor is euphoric when he thinks he has proven CH, and goes ahead with the hierarchy of the Alephs. During depressive periods he sits by a river—a feature of much German romantic literature (one cannot help thinking of Hölderlin's poems).

The Continuum Hypothesis is more than a leitmotif: it is the symbol of the opera. The formula $2^{\aleph_0} = c$ is written at the center of the stage curtain, and is often quickly flashed onto the wall, a hundred times during some scenes. It is even chalked by Cantor on his violin.

Axel Köhler plays Cantor, a perfect role for this very gifted singer. He sings as a baritone when remembering his youth or the happy periods of his life, and as a counter-tenor when thinking about set theory. When Cantor opposes his enemies, like Kronecker, the singer speaks, while his depressive moods are represented by the sound of the violin he plays. The old mathematics is represented by old mandarins in wigs, sometimes even carrying plaster statues. The new mathematics is represented by four charming Alephs, dancing women who come and take off the wigs of the old teachers. The music is an oratorio with most of the themes inspired by the famous Fugue in B Minor from Bach's 'Well-tempered clavier'. This fugue is also well-known to musicians because it uses all twelve half-tones, and it inspired Schoenberg. Grünauer's post-serial style is still quite popular among musicians in Germany. The orchestra, led by Roger Epple, a popular director there, received strong applause.

The staging of the opera is quite elaborate, with part of the orchestra at the rear, sometimes visible, with a part

[1]Simplicius quoting Anaximander in "Commentaries of Aristotle's Physics," 24,13.
[2]Woodin, Hugh W., The continuum hypothesis, Parts 1 and 2, *Notices of the AMS*, vol. 48, 2001.
[3]cf. Reviel Netz and William Noel, *The Archimedes Codex*, May 2007, Weidenfeld & Nicolson.
[4]Stern, Manfred, Memorial Places of Georg Cantor in Halle, *The Mathematical Intelligencer*, v.10, n.3, 1988, 48–49.